Resistive-ballooning-mode characteristics in the tokamak edge region

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The stability characteristics of resistive ballooning modes are examined near a tokamak edge region by including the effects of resistivity gradients. It is shown that the growth rates of electrostatic and Δ'-driven resistive ballooning modes are enhanced, due to parity-mixing effects arising through a linear coupling to rippling modes.

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Recent observations on the L-H transitions [1–3] (i.e., from low to high confinement) and edge turbulence [4,5] in tokamaks have stimulated considerable interest in resistive-ballooning modes. Since the edge region is known to have steep gradients in density and temperature [3], particularly in the vicinity of the separatrix or a magnetic diverter, resistive-ballooning modes are naturally expected to grow there. There is also some experimental evidence (from measurements of radial flux that display large poloidal symmetry [6]) that these modes could be of potential importance for explaining the overall degradation of confinement in tokamaks. A large number of studies have therefore been devoted to various refinements of the theory of resistive-ballooning modes. These include neoclassical viscosity effects [7,8], parallel ion sound compression [8], diamagnetic drift [9], etc. In this paper we consider the effect of the resistivity gradient on the characteristics of the fast resistive-ballooning mode and find that the growth rates of electrostatic and Δ'-driven modes can get enhanced. Thus, although the rippling mode may not directly contribute to the edge fluctuations (as has been shown before [10]), they might still play an interesting role through their contribution to the evolution of the resistive-ballooning modes. We find that the principal physical mechanism responsible for the modification of the stability properties of the ballooning modes arises through the mixing of its parities brought about by the new linear coupling terms.

Consider a plasma edge region where the electron temperature is typically quite low ($T_e < 100$ eV) and the density fairly high. Hence the resistive (magnetohydrodynamics) description becomes appropriate for the plasma behavior there and the dynamical equations for the resistive-ballooning modes can be easily derived from them [11] by adopting the ballooning-mode representation,

$$Q(r,\theta,\zeta)=\sum_{l=-\infty}^{l=\infty}\hat{Q}(\theta+2\pi l)\exp\{i[nq'x(\theta+2\pi l)+n(q_0\theta-\zeta)]\}\ , \quad (1)$$

where r,θ represent polar flux coordinates, ζ is the toroidal angle, Q is any physical quantity, $x=r-r_{mn}$, r_{mn} is the mode-rational surface, $q_0=q(r_{mn})=m/n$, $q(r)=rB_0/RB_\theta(r)$, $q'=dq/dr$, and m and n are the poloidal and toroidal mode numbers, respectively. The model equations are

$$\frac{\partial}{\partial\theta}(1+s^2\theta^2)A_\parallel=i\sigma A_\parallel+\left[\alpha(\cos\theta+s\theta\sin\theta)-\frac{\alpha^2}{2}\right]P+\left[\frac{s\omega}{\omega_A}\right]^2(1+s^2\theta^2)\phi\ , \quad (2)$$

$$\frac{\partial\phi}{\partial\theta}+\frac{\omega_R}{\omega}\phi=-A_\parallel\left[1+\frac{i\nu_R}{\omega}(1+s^2\theta^2)\right]\ , \quad (3)$$

$$\left[1+\left[\frac{\omega_s}{s\omega}\right]^2\frac{\partial^2}{\partial\theta^2}\right]P=-\left[\frac{\omega_s}{s\omega}\right]^2\frac{\partial A_\parallel}{\partial\theta}+\phi\ , \quad (4)$$

where $\sigma=(4\pi qR/cB_0k_\theta)(dJ_{\parallel 0}/dr)$, $\alpha=-8\pi q^2Rp_0'/B_0^2$, $p_0'=dP_0/dr$; the perturbed pressure P is expressed in the units of $(k_\theta p_0'/B_0\omega)$, $\omega_A=sV_A/qR$, $\omega_s=sC_s/qR$, $C_s=\sqrt{T_e/M}$, $s=d\ln q/d\ln r$, $\omega_R=-(k_\theta qRcJ_{0\parallel}/B_0)d\eta_0/dr$, and $\nu_R=\eta_0c^2k_\theta^2/4\pi$. Here $A_\parallel$ is the component of vector potential along the equilibrium magnetic field, and ϕ is the electrostatic potential. Equations (2), (3), and (4), respectively, describe the quasineutrality condition, the modified Ohm's law, and the equation of state. In driving Eq. (3) the resistivity convection law (namely, $d\eta/dt=0$) has been used. In Eqs. (2) and (3), the parity mixing arises due to the terms $\sigma A_\parallel$ and $(\omega_R/\omega)\phi$, which are, respectively, the driving terms for $m=2$ type resistive tearing and rippling modes. In the plasma edge region, the term proportional to ω_R is dominant and we will confine ourselves to the study of the effect of this parity-mixing term on resistive-ballooning modes. In particular, we will consider the linear stability characteristics of fast growing resistive-ballooning modes, in the limit, $\omega\gg\omega_s/s$. This frequency regime is the most relevant from an experimental point of view for the edge region. Hence terms involving $(\omega_s/s\omega)^2$ will be neglected in Eq. (3). It may, however, be pointed out that the inclusion of such terms can, in general, lead to a reduction of growth rates due to a coupling to acoustic waves. Our model equations also do not take into account the effect of parallel thermal conductivity on the ballooning modes, which is justified as long as the mode width is small compared to the thermal diffusivity width, x_t. The latter is defined by $L_s\sqrt{\gamma/k_\theta\sqrt{\chi_\parallel}}$, where L_s is the shear length, $k_\theta=m/r$, $\chi_\parallel$ is the parallel thermal conductivity, and γ is the growth rate of the rippling mode. Near the mode-rational surface $x=0$ this approximation is quite good and thermal effects can be neglected. Likewise the rippling mode driving source is simply modeled by the resistivity gradient term, ignoring the contribution of the impurity ion gradient terms which could dominate when ∇Z_{eff} gets significant. However, the effect of this term would not qualitatively change our results, and the simple driving term captures the essential ingredient of the parity-mixing effect.

Under the above approximations, Eqs. (2)–(4) become simplified for obtaining analytical solutions in the resistive layer. The ballooning mode driving term introduces two scale lengths, $\theta \sim 1$ and $\theta \sim 1/s$, which can be exploited for a multiple scale analysis of the equations. It is also convenient [12] to express $\phi = \phi_0(z) + \phi_c(z)\cos\theta + \phi_s \sin\theta$, where $z = s\theta$, $\phi_{c,s}(z) \ll \phi_0(z)$. Substituting this expression for ϕ in Eqs. (2)–(4) and following the usual averaging procedure [12], the poloidal flux surface-averaged equation in ϕ_0, is given by

$$\frac{d}{dz}\left[f(z)\left\{\frac{d\phi_0}{dz} + \mu\phi_0\right\}\right] + (1+z^2)\left[\frac{\omega^2}{\omega_A^2} + \frac{i\nu_R}{\omega}\frac{\alpha_0^2}{2s^2}\right]\phi_0 = 0 , \quad (5)$$

where $\alpha_0^2 = \alpha^2/(1+\omega_R^2/\omega^2)$, $\mu = \omega_R/s\omega$, and $f(z) = (1+z^2)/\{1+i\nu_R(1+z^2)/\omega\}$. Equation (5) is the eigenvalue equation governing the modified resistive-ballooning-mode characteristics in the resistive layer. It must be noted that the condition $|i\nu_R/\omega|z^2 \gg 1$ defines the domain of validity of localized electrostatic resistive-ballooning modes, while regions in z space typified by $|i\nu_R/\omega|z^2 \leq 1$ describe the evolution of resistive modes (Δ'-driven modes) in which magnetic field perturbations play a crucial role. Clearly, in the absence of linear coupling to rippling modes (i.e., $\mu = 0$), the dispersion relation for growing resistive-ballooning modes can be obtained from Eq. (5) by demanding that the eigensolutions be bounded at $z = \pm\infty$. This convergence condition yields (for $|i\nu_R/\omega|z^2 \gg 1$)

$$\omega^3 = -i\nu_R\omega_A^2\frac{\alpha^2}{2s^2} . \quad (6)$$

Similarly, neglecting the driving term for the resistive-ballooning mode [namely, the $(\alpha^2/2s^2)$ term], Eq. (5) leads to the electrostatic rippling mode dispersion relation [13] (for $|i\nu_R/\omega|z^2 \gg 1$),

$$\omega^5 = e^{i\pi/2}\left[\frac{\omega_R}{s}\right]^4\left[\frac{\omega_A^2}{\nu_R}\right] . \quad (7)$$

For the case when $\omega_R \neq 0$, we shall first consider the electrostatic localized mode within the resistive layer for $z \gg 1$ [in this limit, $f(z)$ can be nearly treated as a constant]. Then Eq. (5) reduces to

$$\frac{d^2\phi_1}{dz^2} + \left[\lambda\nu z^2 - \frac{\mu^2}{4}\right]\phi_1 = 0 , \quad (8)$$

where $\phi_1 = \phi_0\exp(+\mu z/2)$, $\lambda = (\omega^2/\omega_A^2 + \nu\alpha_0^2/2s^2)$, and $\nu = (i\nu_R/\omega)$. From the definition of α_0, it is seen that the ballooning mode driving term, $\alpha_0^2/2s^2$ in λ, is modified due to the rippling parameter ω_R. Equation (8) is a parabolic cylinder equation, and the dispersion relation for the modified resistive-ballooning mode can be derived by demanding that the solutions be bounded at $z = \pm\infty$. Thus we have

$$\left[\frac{\omega}{\omega_A}\right]^2 + \frac{i\nu_R}{\omega}\frac{\alpha_0^2}{2s^2} = \frac{i}{\nu_R}\frac{\omega_R^4}{(2s)^4\omega^3} , \quad (9)$$

in which the right-hand side represents the new contribution due to linear coupling of rippling modes with ballooning modes under discussion. Equation (9) admits seven roots, in general, for different values of the pressure gradient parameter (α) and the resistivity gradient term, ω_R, and out of which only those satisfying the convergency criterion are physically admissible modes. It is clear from Eq. (9) that the limits of the electrostatic rippling mode [Eq. (7)] and fast resistive-ballooning mode [Eq. (6)] can be easily recovered by setting either the pressure gradient term (α) or the resistivity gradient term (ω_R) to zero. To study the influence of the rippling term on the resistive-ballooning mode, we have solved Eq. (9) numerically. Of the many roots of Eq. (9) we systematically track the fast resistive-ballooning mode and study its growth rate as a function of the rippling parameter. The results are displayed in Fig. 1, where the growth rate (normalized to the usual electrostatic resistive-ballooning-mode growth rate in the absence of coupling to the rippling mode) is plotted against the rippling parameter P_R [where $P_R = \omega_R/(\nu_R^{1/3}\omega_A^{2/3})$ is a measure of the resistivity gradient]. We find that the growth rate of the fast resistive-ballooning mode (electrostatic type) is enhanced as a function of the rippling parameter, over a wide range of the parameter L_F [we display the results for two values of $L_F = 10^8$ and 10^3, respectively, where $L_F = (2\omega_A S/\nu_R\alpha)^{4/3}1/(2S)^4$].

We next examine the Δ'-driven modes corresponding to the frequency regime $\omega > \omega_s/s$. It is convenient to rewrite Eq. (5) in terms of a new variable, $J = z^2 A_\parallel$.

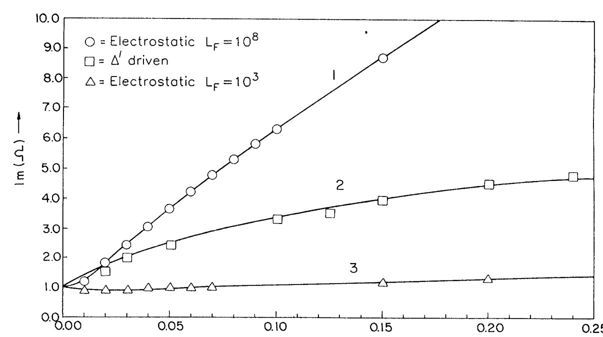

FIG. 1. Plot of the growth rate Im(Ω) as a function of P_R [where $P_R = \omega_R/(\nu_R^{1/3}\omega_A^{2/3})$ is a measure of the resistivity gradient]. The growth rates are normalized to the electrostatic-type resistive-ballooning-mode growth rate for curves 1 and 3 and to the Δ'-driven mode for curve 2 and $L_F = (2\omega_A S/\nu_R\alpha)^{4/3}1/(2S)^4$.

Thus, Eq. (5) for $z^2 \gg 1$ becomes

$$\frac{d}{dz}\left[\frac{1}{z^2}\frac{dJ}{dz}\right]+\frac{\mu}{z^2}\frac{dJ}{dz}+\lambda\left[\frac{1}{z^2}+\nu\right]J=0 . \qquad (10)$$

Equation (10) is not amenable to exact analytical solutions. Among the approximate techniques available to solve this equation, the methods of matched asymptotic expansion and variational procedure are commonly employed. We will obtain a dispersion relation from Eq. (10) by using the variational technique [14]. Setting $J=\exp(-\mu z/2)J_1$, Eq. (10) becomes

$$\frac{d}{dz}\left[\frac{1}{z^2}\frac{dJ_1}{dz}\right]+\left[\frac{1}{z^2}\left[\lambda-\frac{\mu^2}{4}\right]+\frac{\mu}{z^3}+\lambda\nu\right]J_1=0 . \qquad (11)$$

This equation admits variational treatment, in that it can be obtained from a functional,

$$K=\int_{-\infty}^{+\infty}\left\{-\frac{1}{z^2}\left[\frac{dJ_1}{dz}\right]^2+J_1^2\left[\frac{1}{z^2}\left[\lambda-\frac{\mu^2}{4}\right]+\frac{\mu}{z^3}+\lambda\nu\right]\right\}dz . \qquad (12)$$

Choosing a simple trial function of the form $\exp(-\Lambda z^2/2)$ with real $(\Lambda)>0$, we solve the equations $K(\Lambda)=0$ and $\partial K/\partial\Lambda=0$ to obtain the appropriate dispersion relation. Note that the Δ' term is conveniently introduced in this procedure by the prescription [14] of replacing the $1/z^2$ term with $1/z^2-\delta(z)/\Delta'$, where $\delta(z)$ is the Dirac δ function and Δ' is the conventional stability parameter representing the jump in the logarithmic derivative of $A_{\parallel}$ across the resistive layer. The dispersion relation we obtain is

$$\left[\lambda-\frac{\mu^2}{4}\right]^4+\frac{\pi^2}{27}(4\Delta')^4(\lambda\nu)^3=0 . \qquad (13)$$

In the above, setting $\mu=0$, one recovers the dispersion relation for the usual Δ'-driven resistive-ballooning mode. The μ terms represent the coupling of the resistive-ballooning mode to the rippling mode. Equation (20) is difficult to solve analytically except under drastic simplifying assumptions. We have therefore chosen to solve it numerically and study the influence of the rippling parameter on the fast-ballooning-mode branch. The results are displayed in Fig. 1, where we see that the effect is once again an enhancement of the growth rate. Thus both electrostatic and Δ'-driven modes are destabilized by the presence of resistivity gradient coupling. At the tokamak plasma edge where the resistivity gradients are fairly strong, the linear evolution characteristics of the resistive-ballooning modes can therefore get significantly modified.

In summary, we have investigated the stability characteristics of fast resistive-ballooning modes in the presence of linear coupling to rippling modes. Within the framework of our simple model we find that the growth rates of both electrostatic and Δ'-driven resistive-ballooning modes can get significantly enhanced due to the presence of parity-mixing terms. Of course one needs to carry out a nonlinear analysis to ascertain the final behavior of these modes, but the present calculations demonstrate in a simple way the influence of resistivity gradient terms in the linear theory of resistive-ballooning modes. In view of their importance in the edge region, such effects should therefore be incorporated in any detailed physical model of the resistive-ballooning mode.

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